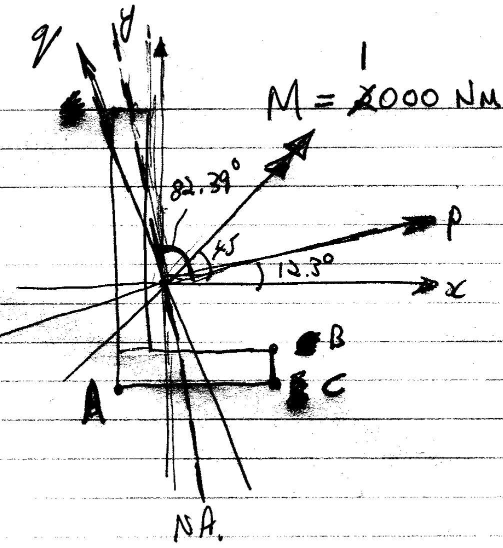
MM2MS2 1112.

Q1.

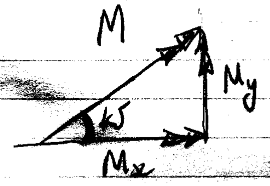


$I_p = 19.25 \cdot 10^5 \text{ mm}^4$

$I_q = 1.65 \cdot 10^5 \text{ mm}^4$

$\theta = 12.3^\circ \quad \sin\theta = 0.213$

$\cos\theta = 0.977$



$M_x = M\cos 45 = 707.1 \text{ Nm}$

$M_y = M\sin 45 = 707.1 \text{ Nm}$

$M_p = M\cos(45 - 12.3)$
$= \underline{841.5} \text{ Nm}$

$M_q = M\sin(45 - 12.3)$
$= \underline{540.2 \text{ Nm}}$

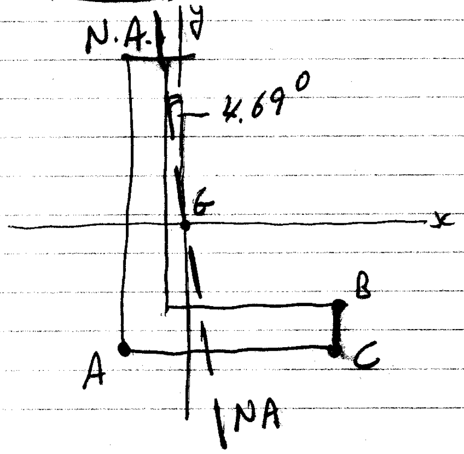
Neutral Axis

$$\alpha = \tan^{-1}\left(\frac{q}{p}\right) = \tan^{-1}\left(\frac{M_q}{M_p}\cdot\frac{I_p}{I_q}\right) = \tan^{-1}\left(\frac{540.2}{841.5}\cdot\frac{19.25}{1.65}\right)$$

$= \underline{82.39^\circ}$ from p-axis

$= \underline{94.69^\circ}$ from x-axis

Sketch



Q1 (cont)

Maximum stress occurs furthest away from Neutral Axis

ie. at A, B, or C.

Co-ordinates $\quad p = x\cos\theta + y\sin\theta \qquad \theta = 12.3$

$= \underline{0.977x + 0.213y} \qquad \cos\theta = 0.977, \quad \sin\theta = 0.213$

$q = -x\sin\theta + y\cos\theta$

$= \underline{0.977y - 0.213x}$

Point	x	y	p mm	q mm
A	-11.67	-41.67	-20.27	-38.22
B	38.33	-31.67	30.70	-39.11
C	38.33	-41.67	28.57	-48.88

Bending Stress $\quad \sigma = \dfrac{M_p q}{I_p} - \dfrac{M_q p}{I_q}$

$= \underline{4.37.10^8 q - 32.73.10^8 p}$

A $\quad \sigma_{bA} = 49.64$ MPa tensile

B $\quad \sigma_{bB} = 117.6$ MPa compressive

C $\quad \sigma_{bC} = 114.8$ MPa compressive

Max Bending stress = 117.6 MPa Compressive

at Position B

ie. position furthest from N.A.

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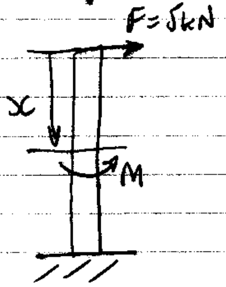
Q2

Bending



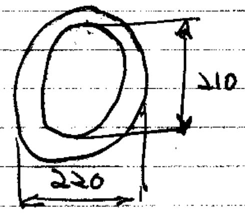
$M = Fx$

$$U_B = \int_0^L \frac{M^2}{2EI}\,dx$$

$$= \int_0^L \frac{F^2x^2}{2EI}\,dx$$

$$= \frac{F^2}{2EI}\left[\frac{x^3}{3}\right]_0^L$$

$$= \underline{\underline{\frac{F^2L^3}{6EI}}} \quad [3\text{ marks}]$$



$$I = \frac{\pi}{64}\left[220^4 - 210^4\right]$$

$$= 1.952 \cdot 10^7\ \text{mm}^4$$

$$= \underline{1.952 \cdot 10^{-5}\ \text{m}^4}$$

$$J = 2I = \underline{3.904 \cdot 10^{-5}\ \text{m}^4}$$

[5 marks]

$$\therefore U_B = \frac{(5 \cdot 10^3)^2 \cdot (3)^3}{6 \cdot 40 \cdot 10^9 \cdot 1.952 \cdot 10^{-5}} = \underline{\underline{144.1}}\ \text{Joules}$$

[2 marks]

Torsion $\quad T = 10\,\text{kNm}$ constant.

$$U_T = \int_0^L \frac{T^2}{2GJ}\,dx$$

$$= \frac{T^2}{2GJ}\left[x\right]_0^L$$

$$= \underline{\underline{\frac{T^2L}{2GJ}}} \quad [3\text{ marks}]$$

$$\therefore U_T = \frac{(10 \cdot 10^3)^2 \cdot 3}{2 \cdot 15 \cdot 10^9 \cdot 3.904 \cdot 10^{-5}} = \underline{\underline{256.1\ \text{Joules}}}$$

[2 marks]

$\therefore$ Total Strain Energy $\quad U_{tot} = 144.1 + 256.1 = \underline{\underline{400.2\ \text{Joules}}}$

[2 marks]

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Q2(cont)

$$U_{tor} = \frac{F^2 L^3}{6EI} + \frac{T^2 L}{2GJ}$$

Bending

$$u_F = \frac{\partial U_{tor}}{\partial F} = \frac{2FL^3}{6EI}$$

$$= \frac{2 \cdot 5.10^3 \cdot 3^3}{6 \cdot 40.10^9 \cdot 1.95.10^{-5}}$$

$$= \underline{\underline{57.6 \text{ mm}}} \quad [8 \text{ marks}]$$

Torsion

$$u_T = \frac{\partial U_{tor}}{\partial T} = \frac{2TL}{2GJ}$$

$$= \frac{2 \cdot 10.10^3 \cdot 3}{2 \cdot 15.10^9 \cdot 3.904.10^{-5}}$$

$$= 0.0512 \text{ rad}$$

$$= \underline{\underline{2.94^0}} \quad [8 \text{ marks}]$$

Q3

$$\sigma_r = A - \frac{B}{r^2} - \frac{\rho w^2 (3+\nu) r^2}{8}$$
$$\sigma_\theta = A + \frac{B}{r^2} - \frac{\rho w^2 (1+3\nu) r^2}{8}$$

At r = 100mm or (0.1m), σ_r=0

$$0 = A - \frac{B}{0.1^2} - \frac{7850x10^{-9}}{8}\left(5000\frac{2\pi}{60}\right)^2 (3+0.3) 0.1^2 x10^{-3}$$

i.e.

$$0 = A - \frac{B}{0.01} - 8.877x10^6 \qquad (1)$$

At r = 500mm (or 0.5m), σ_r=0

$$0 = A - \frac{B}{0.5^2} - \frac{7850}{8}\left(5000\frac{2\pi}{60}\right)^2 (3+0.3) 0.5^2$$
$$0 = A - \frac{B}{0.25} - 2.22x10^8 \qquad (2)$$

Subtract (1) from (2)

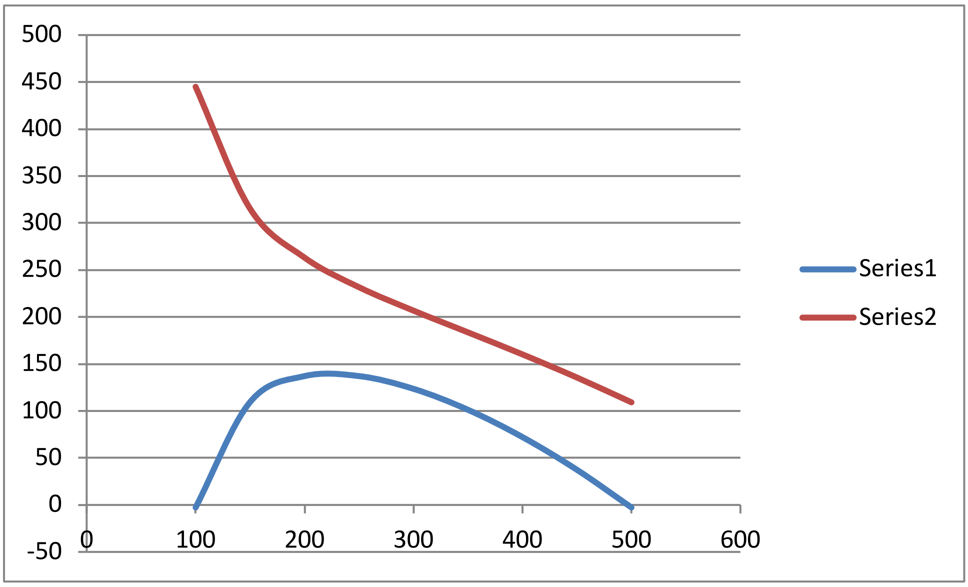
B = 2.22 x10^6
A = 2.28x 10^8

[20 marks]

Hence

$$\sigma_r = 2.28x10^8 - \frac{2.22x10^6}{r^2} - \frac{7850}{8}\left(5000\frac{2\pi}{60}\right)^2 (3+0.3) r^2$$
$$\sigma_\theta = 2.28x10^8 + \frac{2.22x10^6}{r^2} - \frac{7850}{8}\left(5000\frac{2\pi}{60}\right)^2 (3+0.3) r^2$$

r	σ_r MPa	σ_h MPa
0.1	-2.71257	444.9281
0.15	109.489	315.2405
0.2	137.1079	263.1413
0.25	137.1079	231.6666
0.3	123.545	206.7603
0.35	101.2356	183.6063
0.4	72.19124	160.1927
0.45	37.37313	135.5586
0.5	-2.71257	109.1976

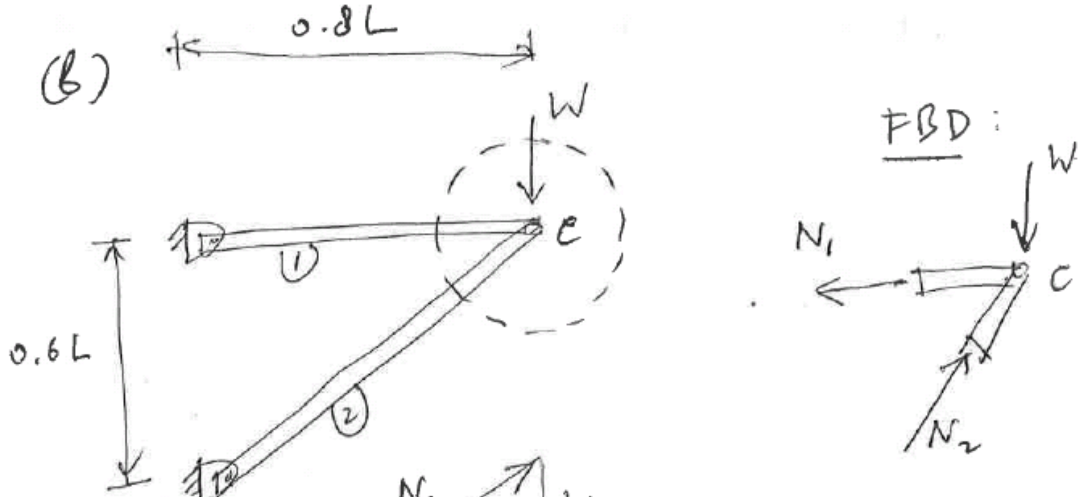

500
450
400
350
300
250
200
150
100
50
0
-50
0
100
200
300
400
500
600
Series1
Series2

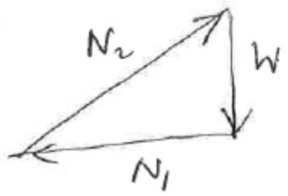
[13 marks]

Q4

(a) $P_{cr} = \pi^2 EI/L^2$ [4 marks]

(b)





$$N_2 = \frac{5}{3} W$$

at max capacity:

$$N_2 = P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\frac{5}{3} W = \frac{\pi^2 EI}{L^2} \Rightarrow \therefore W = 0.6 \frac{\pi^2 EI}{L^2}$$

[18 marks]

(c) $$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 E_2 I_2}{L^2}$$

$$I_2 = \frac{E}{E_2} I = 3I.$$

$$E_2 = E/3$$

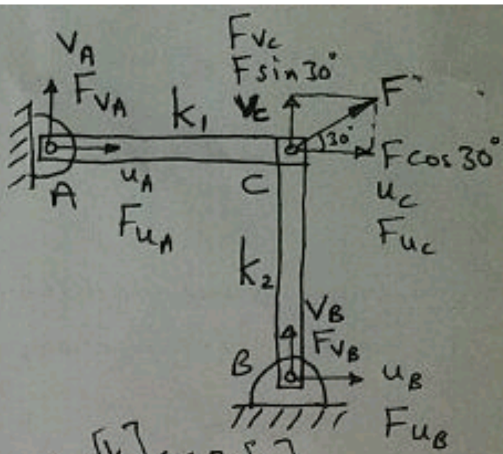
$$I \propto d^4$$

$$\therefore \frac{d_2}{d} = \sqrt[4]{3} = 1.32$$

[11 marks]

5.

Solution for Q2



(a) $[k_1]_G = \frac{AE}{L}\begin{bmatrix}1&0&-1&0\\0&0&0&0\\-1&0&1&0\\0&0&0&0\end{bmatrix}$

$[k_2]_G = \frac{AE}{L}\begin{bmatrix}0&0&0&0\\0&1&0&-1\\0&0&0&0\\0&-1&0&1\end{bmatrix}$

from $[k_1]_G$

from $[k_1]_G$ & $[k_2]_G$

$$\begin{Bmatrix}F_{u_A}\\F_{V_A}\\F_{u_c}\\F_{V_c}\\F_{u_B}\\F_{V_B}\end{Bmatrix} = \frac{AE}{L}\begin{bmatrix}1&0&-1&0&0&0\\0&0&0&0&0&0\\-1&0&1&0&0&0\\0&0&0&1&0&-1\\0&0&0&0&0&0\\0&0&0&-1&0&1\end{bmatrix}\times\begin{Bmatrix}\cancel{u_A}\to 0\\\cancel{V_A}\to 0\\u_c\\V_c\\\cancel{u_B}\to 0\\\cancel{V_B}\to 0\end{Bmatrix}$$

Matrix Equation

from $[k_2]_G$

∴ stiffness matrix of the structure $= \frac{AE}{L}\begin{bmatrix}1&0&-1&0&0&0\\0&0&0&0&0&0\\-1&0&1&0&0&0\\0&0&0&1&0&-1\\0&0&0&0&0&0\\0&0&0&-1&0&1\end{bmatrix}$ [13 marks]

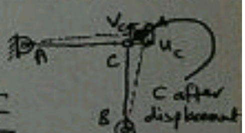
(b) (1) From Matrix Equation (above): $F_{u_c} = \frac{AE}{L}\times(+u_c)$

But F_{u_c} is $F\cos 30°$ (from given force) ∴ $u_c = \frac{L}{AE}F\cos 30° = \frac{\sqrt{3}}{2}\frac{LF}{AE}$

∴ Horizontal displacement at C $= u_c = \frac{\sqrt{3}}{2}\frac{LF}{AE}$

Similarly $F_{V_c} = \frac{AE}{L}\times(+V_c)$, but $F_{V_c} = F\sin 30°$

∴ Vertical displacement at C $= V_c = \frac{L}{AE}F\sin 30° = \frac{1}{2}\frac{LF}{AE}$ [10 marks]



(2) From Matrix Equation: $F_{u_A} = \frac{AE}{L}\times(-u_c) = \frac{AE}{L}\times\left(-\frac{\sqrt{3}}{2}\frac{LF}{AE}\right) = \frac{-\sqrt{3}}{2}F$

$F_{V_A} = 0$

∴ Reaction on support A $= \frac{\sqrt{3}}{2}F$ ($\overrightarrow{\text{horizontal}}$)

$F_{u_B} = 0$, $F_{V_B} = \frac{AE}{L}\times(-V_c) = \frac{AE}{L}\times\left(-\frac{1}{2}\frac{LF}{AE}\right) = \frac{-F}{2}$

∴ Reaction on support B = F/2 (vertical ↑) [10 marks]